MATH 728
FALL 2004
HOMEWORK 6

Due Wednesday October 20, 2004.

1. Let R be commutative with identity and M and $R-$module.

a) (5 pt) Show if M is free, then M is flat.

b) (5 pt) Show that if M is projective, then M is flat.

2. Let $S \subseteq \mathbb{Z}$ be a multiplicatively closed subset that does not contain 0. Consider the $\mathbb{Z}-$module $\mathbb{Z}_S = \{n/s | n \in \mathbb{Z}, s \in S\}$.

a) (5 pt) Compute $\mathbb{Z}_S \otimes_{\mathbb{Z}} \mathbb{Z}_{p^a}$ where $p \in \mathbb{Z}$ is a nonzero prime.

b) (5 pt) Generalize your result from part a) by computing $\mathbb{Z}_S \otimes_{\mathbb{Z}} A$ where A is any finite abelian group.

c) (5 pt) Generalize the results from a) and b) by computing $\mathbb{Z}_S \otimes_{\mathbb{Z}} G$ where G is any finitely generated abelian group.

3. (5 pt) Show that $\mathbb{Z}_m \otimes_{\mathbb{Z}} \mathbb{Z}_n \cong \mathbb{Z}_{\gcd(m,n)}$.

4. (5 pt) Show that if P and Q are projective $R-$modules, then $P \otimes_R Q$ is a projective $R-$module. Is the converse true?

5. (5 pt) Show that $\mathbb{Q}$ is a flat $\mathbb{Z}-$module which is not projective.