

MATH 728
FALL 2004
HOMEWORK 7

Due Monday November 29, 2004.

1. Find the canonical forms (the rational canonical form, primary rational canonical form and Jordan canonical form if possible) for the following matrices over \mathbb{Q} :

a) (5 pt)
$$\begin{bmatrix} 3 & -4 & -6 & -9 & 0 \\ -1 & 7 & 8 & 11 & -1 \\ 1 & 0 & 1 & -1 & -1 \\ 0 & -3 & -4 & -4 & 1 \\ 1 & -1 & -2 & -3 & 1 \end{bmatrix}$$

b) (5 pt)
$$\begin{bmatrix} 0 & 2 & 0 & -2 & 3 \\ 4 & -3 & -2 & -1 & -1 \\ -1 & 4 & 0 & -2 & 5 \\ 1 & 2 & 0 & -3 & 3 \\ -2 & 2 & 1 & 0 & 1 \end{bmatrix}$$

2. (5 pt) Show that an $n \times n$ matrix (A) over a field \mathbb{F} is similar to a diagonal matrix if and only if there is a basis of \mathbb{F}^n consisting of eigenvectors of A .
3. (5 pt) Let $0 \neq p \in \mathbb{Z}$ be a prime and \mathfrak{M} be the category of finite abelian p -groups. Compute $K_0(\mathfrak{M})$.
4. (5 pt) Let \mathfrak{N} be a category that is closed under countable direct sum (that is, if $\{C_i\}_{i=1}^{\infty}$ is a collection of objects of \mathfrak{N} , then $\bigoplus_{i=1}^{\infty} C_i$ is also an object in \mathfrak{N}). Show that $K_0(\mathfrak{N}) = 0$.