MATH 772
SUMMER 2006
HOMEWORK 4

Due Monday, July 24, 2006.

1. Find the class group for each quadratic ring of integers below.
   a) (5 pt) $\mathbb{Z}[\sqrt{-14}]$.
   b) (5 pt) $\mathbb{Z}[\sqrt{-10}]$.
   c) (5 pt) $\mathbb{Z}[\frac{1+\sqrt{-23}}{2}]$.
   d) (5 pt) $\mathbb{Z}[\sqrt{-21}]$
   e) (5 pt) $\mathbb{Z}[\frac{1+\sqrt{-163}}{2}]$.

2. (5 pt) Find the smallest positive square-free integer, d, such that the ring of integers of the field $\mathbb{Q}[\sqrt{d}]$ is not a UFD.

3. (5 pt) Explicitly show for quadratic fields that a prime is ramified if and only if it divides the discriminant.

4. Consider the field $\mathbb{Q}(\alpha)$ where α is a root of $x^5 - x^3 + 1$. You may assume that the ring of integers of $\mathbb{Q}(\alpha)$ is $\mathbb{Z}[\alpha]$.
   a) (5 pt) Find the number of real and complex embeddings of $\mathbb{Q}(\alpha)$ into $\mathbb{C}$.
   b) (5 pt) Find the discriminant of the field $\mathbb{Q}(\alpha)$.
   c) (5 pt) Find the class group of the ring $\mathbb{Z}[\alpha]$.
   d) (5 pt) Determine how the ramified primes factor in $\mathbb{Z}[\alpha]$.
   e) (5 pt) Show that there is an element of norm 27 and of norm 9, but no element of norm 3.