(1) Show that there is no simple group of order 96.

(2) Show that if \( \alpha \) is a root of the polynomial \( f(x) = x^5 - x^3 + 1 \) and \( F = \mathbb{Q}(\alpha) \) then \( F \) is not Galois over \( \mathbb{Q} \).

(3) Let \( R \) be a commutative ring with identity and \( P \) a projective \( R \)-module and \( N \) a free \( R \)-module. Show that \( P \otimes_R N \) is also a projective \( R \)-module.

(4) Show any finite subgroup of the multiplicative group of a field is cyclic.

(5) Suppose that \( R \) is a non-Noetherian ring. Show that there is a prime ideal \( \mathfrak{p} \subseteq R \) such that \( \mathfrak{p} \) is not finitely generated.

(6) Suppose that \( G \) is a group and \( x \) is an element with precisely two distinct conjugates (the number of distinct elements in the set \( \{ g^{-1} x g | g \in G \} \) is 2). Show that \( G \) possesses a nontrivial normal subgroup.

(7) Suppose that \( G \) is a finite group and for each prime \( p \) dividing the order of \( G \) the Sylow \( p \)-subgroup is normal. Show that if the order of \( G \) is not divisible by any cube (other than 1), then \( G \) is abelian.

(8) Let \( T \) be a torsion abelian group (that is, every element is of finite order) and let \( \mathbb{Q} \) denote the additive group of rational numbers. Show that \( T \otimes \mathbb{Z} \mathbb{Q} = 0 \).

(9) Consider the following commutative diagram of \( R \)-modules you may assume that \( R \) is commutative with identity and all modules are unitary) with exact rows:

\[
\begin{array}{cccccc}
0 & \rightarrow & A_1 & \rightarrow & A_2 & \rightarrow & A_3 & \rightarrow & 0 \\
& & \downarrow f & & \downarrow g & & \downarrow h & \\
0 & \rightarrow & B_1 & \rightarrow & B_2 & \rightarrow & B_3 & \rightarrow & 0
\end{array}
\]

Show that if \( f \) and \( h \) are one to one, then so is \( g \).
(10) Find all monic irreducible polynomials of degree 2 over the field \( \mathbb{F}_3 \) of three elements. For each irreducible \( p(x) \) that you find, determine the ring structure of the quotient ring \( \mathbb{F}_3[x]/(p(x)) \). How many isomorphism classes are there?