Friday, October 24

8:00 AM Bruce M Olberding, An exceptional class of stable rings.

A commutative $R$ is stable if every regular ideal $I$ of $R$ is invertible when viewed as an ideal over its ring of endomorphisms. A classification of Noetherian stable rings can be deduced from work of J. Lipman; J. Sally and W. Vasconcelos; and D. Rush. Recently the general case of not-necessarily-Noetherian stable domains has also been explicitly described. A crucial property in describing this class of domains is that “most” quasilocal stable domains arise as pullbacks of strongly discrete valuation domains and one-dimensional stable domains. If $R$ is a quasilocal stable domain that does not arise this way, then we say $R$ is “exceptional.” The reason for this terminology is that according to Lemma 4.2 of my article “On the structure of stable domains,” Comm. Alg 30 (2002), 877-895, such a quasilocal stable domain cannot exist. In this talk I will explain this error, give examples and indicate how some subsequent results on the classification of stable domains must be modified. For example, the above characterization becomes: A quasilocal domain $R$ that is not a field is stable if and only if it arises as a certain pullback of a strongly discrete valuation domain and a one-dimensional stable ring with minimal prime ideal $P$ such that $P^2 = 0$.


An extension $R \subseteq T$ of (commutative) rings is said to satisfy the FIP property if the set of all (unital) $R$-subalgebras of $T$ is finite (D.D. Anderson, D.E. Dobbs, and B. Mullins) and to be minimal if $R \neq T$ and there is no ring strictly between $R$ and $T$. In general, the FIP property is not preserved by the formation of Nagata rings. The following are two positive results on Nagata rings. If $R \subseteq T$ is a finite minimal extension of rings, then so is $R(X) \subseteq T(X)$. If an extension of fields $K \subseteq L$ satisfies FIP, then so does $K(X) \subseteq L(X)$. A characterization of FIP extensions $R \subseteq T$ is obtained in the following two cases: $R$ is an Artinian reduced ring; $T = R[u]$ with $u$ nilpotent. Finally, if $D$ is an integral domain with quotient field $K$, it is proved that $D$ is integrally closed such that $D \subseteq K$ has FIP if and only if $D$ is a strong G-domain with finite spectrum.

9:00 AM Ayman Badawi and Thomas G Lucas*, On $\phi$-Mori rings.

Let $R$ be a commutative ring whose nilradical $\text{Nil}(R)$ is prime and comparable with every ideal of $R$; i.e., $\text{Nil}(R)$ is a divided prime of $R$. Then the nilradical is common prime of $R$ and its total quotient ring $T(R)$. Moreover, there is a natural map $\phi$ from $T(R)$ into $R_{\text{Nil}(R)}$ which maps $\text{Nil}(R)$ onto the nilradical of $R_{\text{Nil}(R)}$. An ideal $I$ of $R$ is said to be $\phi$-divisorial if $\phi(I)$ is a divisorial ideal of $\phi(R)$ and $I$ properly contains $\text{Nil}(R)$. The ring $R$ is said to be a $\phi$-Mori ring if it satisfies the ascending chain condition on $\phi$-divisorial ideals. The following are shown to be equivalent: (1) $R$ is a $\phi$-Mori ring, (2) $\phi(R)$ is a Mori ring, and (3) $R/\text{Nil}(R)$ is a Mori domain. One problem to be explored is the following: “Characterize when the Nagata ring $R(X)$ is a $\phi$-Mori ring.” A (somewhat) complete characterization is given in the case $Z(R) = \text{Nil}(R) \neq (0)$.
9:30 AM Bruce Olberding, Serpil Saydam and Jay Shapiro*, Completions, filtrations and ultrapowers of local domains.

We examine the structure of the ultrapower $R^*$ of a local (Noetherian) ring $(R, M)$ over a free ultrafilter on an index set $I$. We know that $R^*$ is quasilocal with maximal ideal denoted $M^*$. Let $B = \bigcap_{k=1}^\infty (M^*)^k$. Then it was shown in [Gilmer and Heinzer, Infinite products of zero-dimensional commutative rings, Houston J. Math. 21 (1995), 247-259] that if $I$ is countable, there is a natural injection from the completion $\hat{R} \hookrightarrow R^* := R/B$ which, if $R$ has finite residue, is an isomorphism. Using Cohen’s structure theorem we generalize these results to describe $R^*$ in terms of a coefficient ring of $\hat{R}$. In particular, $R^*$ is a complete local ring that is faithfully flat extension of $R$. We then develop the notion of a filtration on a ring which is broad enough to include the order function with respect to an ideal as well as valuations. These can be used to describe certain infinite chains of radical ideals of $R^*$ (which include the ideal $B$ above). They will be prime precisely when $R$ is analytically irreducible.

10:00 AM Sarah Glaz, Extension of the notion of a Prufer domain to rings with zero-divisors.

An integral domain whose finitely generated ideals are invertible is called a Prufer domain. These rings were first defined by Heinz Prufer (1896 - 1934). Since then research into the properties of Prufer domains made them one of the most useful tools of inquiry into the behavior of integral domains. The Prufer domain notion can be generalized to rings with zero-divisors in a number of ways. This talk will discuss several of these generalizations, due to the speaker and other researchers. We will focus on the relations between the various generalizations, and on their usefulness as investigation tools in the theory of general non-Noetherian rings.

10:30 AM Byung Gyun Kang, SFT stability via power series extension.

It is known that the power series ring over a finite dimensional SFT Prufer domain is an SFT ring. Generalizing this result to the infinite dimensional case, we show that the power series ring over an SFT Prufer domain is also an SFT ring.

2:30 PM J-L Chabert, On the Ideal generated by the Values of a Polynomial.

Let $D$ be the ring of integers of a number field $K$ and let $P(X) = \sum_{k=0}^n a_k X^k \in K[X]$. It is well known that, for each maximal ideal $m$ of $D$ with norm $q$, one has:

$$v_m(P(D)) \leq v_m(P(X)) + w_q(\deg P)$$

where $v_m$ denotes the valuation on $K$ associated to $m$, $v_m(P(D)) = \inf \{v_m(P(b)) \mid b \in D\}$, $v_m(P(X)) = \inf \{v_m(a_k) \mid 0 \leq k \leq \deg P\}$ and $w_q(n) = \sum_{k=1}^\infty \lfloor n/q^k \rfloor$. Vajaitu gave another bound for $v_m(P(D))$ in the case where $D=\mathbb{Z}$. We extend this bound to more general cases and in particular to ring of integers of number fields:

$$v_m(P(D)) < v_m(P(X)) + v_m(P)$$

where $\nu_m(P)$ denotes the cardinality of the set $\{k \mid v_m(a_k) = v_m(P(X))\}$.

3:00 PM Neal O Smith, Planar Zero-Divisor Graphs.

Given a commutative ring $R$, one can form a graph $\Gamma(R)$ whose vertices are the nonzero zero-divisors of $R$. Two vertices $x$ and $y$ are adjacent in $\Gamma(R)$ if and only if $xy = 0$. Anderson, Lauve, Livingston and Frazier asked which finite commutative rings have the feature that $\Gamma(R)$ is planar.
We answer this question and further show that if $R$ is an infinite commutative ring with $\Gamma(R)$ planar, then $\Gamma(R)$ must belong to one of three classes of graphs.

3:30 PM K Alan Loper* and Moshe Roitman, The content of a Gaussian polynomial is invertible.

Let $D$ be an integral domain and let $f(x)$ be a nonzero polynomial in $D[x]$. We call the ideal $c(f) =$ (the ideal generated by the coefficients of $f(x)$) the content ideal of $f(x)$. We say that $f(x)$ is Gaussian if $c(fg) = c(f)c(g)$ for every $g(x)$ in $D[x]$. It is well known that if $c(f)$ is an invertible ideal, then $f$ is Gaussian. In this talk we prove the converse.

4:00 PM David E Dobbs* and Brian C Irick, The minimal generating sets of the multiplicative monoid of a finite commutative ring.

For any finite commutative multiplicative monoid $S$ with an element 0 such that $S0 = \{0\} \neq S$, some decompositions of $S$ are given as the disjoint union of a submonoid of $S$ and some prime ideals of some submonoids of $S$. These decompositions lead to an algorithm producing all the minimal generating sets of $S$ in terms of semigroup-theoretic generating sets of minimal prime ideals of some submonoids of $S$ and minimal generating sets of the group of invertible elements of $S$. This algorithm is applied in case $S$ is the multiplicative monoid of a finite nonzero commutative ring $R$. For any such $R$, each application of the algorithm terminates in the same number of steps, namely, the number of prime ideals of $R$, that is, the number of minimal prime ideals of $S$.

4:30 PM Sophie Frisch, Integer-valued polynomials on matrices.

A polynomial $f$ with coefficients in $K$, the quotient field of an integral domain $D$, is called integer-valued on $n \times n$ matrices if, for every $n \times n$ matrix $M$ with entries in $D$, $f(M)$ is again a matrix with entries in $D$. We present some facts concerning separation of points (given matrices $A, B$ when is there an integer-valued polynomial $f$ with $f(A) = 0$ and $f(B) = \text{I}_n$) and interpolation as well as some related results from linear algebra over commutative rings concerning minimal polynomials and null ideals of matrices.


We recall several results of zero divisor graphs of commutative rings. We then examine the preservation of diameter and girth of the zero divisor graph under extension to polynomial and power series rings.

Saturday, October 25

8:00 AM Karl Kattchee, Enumerating the elasticities of Krull domains with divisor class group $\mathbb{Z}_{p^k}$

Given a finite abelian group $G$, define the set

\[ \Upsilon(G) = \{ \rho(R) : R \text{ is a Krull domain with } Cl(R) = G \} , \]

where $\rho$ is the elasticity function. Translating into the language of block monoids, we have

\[ \Upsilon_i(G) = \{ \rho(B(G, S)) : S \text{ is a generating subset of } G \} , \]

where $B(G, S)$ is the monoid of zero-systems of $G$ involving only elements of $S$. If we define $\Upsilon_i(G)$ to be the subset of $\Upsilon(G)$ obtained by adding the condition that the set $S$ have at most $i$ elements,
then the question arises: What is the smallest value \( i \) such that \( \Upsilon(G) = \Upsilon_i(G) \)? Our focus is on the case \( G = \mathbb{Z}_{p^k} \). We show, for example, that \( \Upsilon(\mathbb{Z}_{2^k}) = \Upsilon_2(\mathbb{Z}_{2^k}) \) for \( k = 1, 2, 3, 4, 5, 6 \).

8:30 AM Jack L Maney, On Integral Morphisms.

Recall that a half factorial domain (HFD) \( R \) is an atomic domain where given any collection of irreducibles \( \{ \alpha_1, \alpha_2, \ldots, \alpha_m, \beta_1, \beta_2, \ldots, \beta_n \} \) with

\[
\alpha_1 \alpha_2 \cdots \alpha_m = \beta_1 \beta_2 \cdots \beta_n
\]

we have \( n = m \). In a 1999 paper by J. Coykendall, a generalization of the length map of Zaks, called the boundary map was introduced. In this talk, we look at generalizations of the boundary map, called integral morphisms. Using integral morphisms, we will generalize many of the author’s results about overrings of an HFD.

9:00 AM M. Axtell*, J. Stickles, S. J. Forman and N. Roermsa, U-Factorizations: Rearrangements and Idealizations.

Let \( R \) be a commutative ring with identity. In this talk we examine a type of factorization called a U-factorization. We discuss rearrangements of U-factorizations and extend several results concerning finite factorization properties to U-factorizations. We also explore the close relationship between a U-factorization in a ring \( R \) and a factorization in a related monoid, \( (R/*, *) \), where \( * \) is the associate relationship. Time permitting, we will discuss U-factorizations in idealizations.

9:30 AM Florian Kainrath, On local half-factorial orders.

Let \( R \) be an atomic domain. Then every non unit \( r \) of \( R \) has a factorization \( r = u_1 \cdots u_n \), where the \( u_i \) are atoms of \( R \). The integer \( n \) is called the length of the factorization. \( R \) is called halffactorial, if for any \( r \in R \) all the factorizations of \( r \) have the same length. In this paper we study local orders, i. e. local integral domains \( R \), which are noetherian, one-dimensional, whose integral closure is a finitely generated \( R \)-module, and whose residue field is finite. We give a new criterion, when such domains are halffactorial. As an immediate application of this criterion we obtain a result on the size of the conductor of a halffactorial order, and we show that any ring between a halffactorial order and its quotient field is halffactorial, too. The proof of the above mentioned criterion is mainly based on a result on the product of two subspaces in a finite extension of fields, which is completely analogous to a result of Kneser on the sumset of two finite sets in an abelian group.

10:00 AM Stephen J McAdam* and Richard G Swan, Comaximal Factorization of Ideals.

Emmy Noether showed that any ideal \( I \) in a Noetherian ring has a unique complete comaximal factorization. We considerably weaken the Noetherian hypothesis and supply a very easy proof. We relate this to certain interesting partitions of \( V(I) \).

10:30 AM J. I. Garcia-Garcia*, P. A. Garcia-Sanchez and J. C. Rosales, Every positive integer is the Frobenius number of a numerical semigroup with three generators.

Let \( n_1, \ldots, n_p \) be positive integers with greatest common divisor (gcd for short) one. Then it is not hard to show that there are finitely many nonnegative integers that cannot be expressed as a nonnegative integer linear combination of \( n_1, \ldots, n_p \). The largest nonnegative integer fulfilling this condition is usually known as the Frobenius number of \( n_1, \ldots, n_p \) and it will be denoted by \( F(n_1, \ldots, n_p) \). The problem of determining \( F(n_1, \ldots, n_p) \) appears in the literature as the Frobenius coin-exchange problem. For \( p = 2 \), Sylvester proved that \( F(n_1, n_2) = n_1 n_2 - n_1 - n_2 \). No general
formula has been found so far for the case \( p \geq 3 \). Moreover, as Curtis shows, there is no closed formula of a certain type for \( p = 3 \). If we focus our attention on this case, then we can think of \( F \) as a correspondence that maps three relatively prime integers \( n_1, n_2, n_3 \) to a nonnegative integer \( F(n_1, n_2, n_3) \). In this work we prove that this map is surjective, that is, for every positive integer \( g \) there exist positive integers \( n_1, n_2, n_3 \) such that \( F(n_1, n_2, n_3) = g \).

2:30 PM Evan G Houston* and Muhammad Zafrullah, *On UMV-domains.*

Call an integral domain \( R \) a UMV-domain if every upper to zero in \( R[X] \) is a maximal \( v \)-ideal (maximal divisorial ideal). We show that if \( R \) is a UMV-domain and \( P \) is a divisorial prime ideal of \( R \), then \( R_P \) has Prüfer integral closure. We also show that the UMV-property is not stable under localization.

3:00 PM Wolfgang Hassler, *Factorizations with successive lengths in one-dimensional local domains.*

Let \( D \) be an atomic domain and \( 0 \neq a \in D \) a non-unit. Two integers \( k < l \) are called successive lengths of \( a \) if

\[
L(a) \cap \{ n \in \mathbb{N} \mid k \leq n \leq l \} = \{k, l\},
\]

where

\[
L(a) = \{ n \in \mathbb{N} \mid a \text{ has a factorization into } n \text{ irreducible elements of } D \}
\]

denotes the set of lengths of \( a \). Suppose that \( D \) is a one-dimensional local Noetherian domain. Then it is well known that

\[
\{ l - k \mid 0 \neq a \in D \setminus D^\times, \ k < l \text{ are successive lengths of } a \}
\]

is bounded. (Here \( D^\times \) denotes the group of units of \( D \)). For \( 0 \neq a \in D \setminus D^\times \) and \( n \in \mathbb{N} \) let \( Z_n(a) \) denote the set of factorizations of \( a \) with length \( n \). We are interested in the structure of the sets \( Z_n(a) \) (which are indeed metric spaces in a natural way) and ask the following questions: What is the “relation” between \( Z_k(a) \) and \( Z_l(a) \) if \( k \) and \( l \) are successive lengths of \( a \)? What is the structure of “chains” of factorizations in \( Z_n(a) \) if \( n \in L(a) \)?

3:30 PM Gabriel Picavet, *Localization with respect to endomorphisms.*

We introduce the class of localizable monoids. It contains inverse monoids. Then we define localizations of monoids with respect to localizable submonoids of their monoid of endomorphisms. These constructions can be applied to a category of left modules or to a category of \( A \)-rings. As a result, we are able to invert endomorphisms within the original category, unlike inversive localizations of Cohn's type which need a base change. Some applications to commutative (unital) algebras are given.

4:00 PM Bruce Olberding and A. Serpil Saydam*, *Projective presentations of finitely generated modules with large annihilators.*

Let \( R \) be an integral domain, and let \( A \) be an “almost” finitely presented \( R \)-module such that \( R/\text{Ann}(A) \) is a finite direct sum of quasilocal rings. We characterize the relationship between the modules that can occur as kernels of finite rank projective presentations of \( A \), and we give a criterion for when local isomorphism of torsionless modules implies an equivalence of the two modules up to summands of projective \( R \)-modules.
4:30 PM Tracy Dawn Hamilton, *Unmixedness and the Generalized Principal Ideal Theorem.*

Recent work toward extending the theory of Cohen-Macaulayness to all commutative rings (both Noetherian and non-Noetherian) has led to the definition of weak Bourbaki unmixed rings (wB-unmixed) and weak Bourbaki height-unmixed rings (wB-h-ht-unmixed). In this work we study these unmixedness conditions on rings which satisfy the generalized principal ideal theorem (GPIT) or at least the principal ideal theorem (PIT). This is a natural extension from Noetherian rings since Noetherian rings satisfy GPIT and PIT. There are, however, many rings which satisfy GPIT and/or PIT which are not Noetherian. Among the results are the following: (1) In rings which satisfy GPIT, wB-h-ht-unmixed is equivalent to wB-unmixed. (2) Every unmixed domain (in either sense) satisfies PIT. (3) Locally Cohen-Macaulay rings (which are locally Noetherian and therefore satisfy GPIT) are unmixed. As a corollary to result (2) we also get that a Prufer domain $R$ is wB-h-ht-unmixed if and only if $\dim(R) \leq 1$.

5:00 PM David F Anderson, GyuWhan Chang and Jeanam Park*, *A General Theory Of Splitting Sets.*

Let $S$ be a finite type star operation on an integral domain $D$ and $S$ a saturated multiplicative subset of $D$. We say that $S$ a $g^*$-splitting set if for $0 \neq d \in D$, we can write $d = st$ for some $s \in S$ and $t \in D$ with $(s', t)^* = D$ for all $s' \in S$. In this paper, we generalize some well known results of splitting sets to $g^*$-splitting sets.

5:30 PM James Coykendall and Tridib Dutta*, *Properties of M-almost integrality.*

In this talk we are going to study a type of integrality which is in between almost integrality and ordinary integrality. If $R$ is an integral domain, an element $y$ of the quotient field of $R$ is said to be $m$-almost integral over $R$ if $xy$ in $R$, where $x$ is in $R$, then there exists a positive integer $m$ such that $x^m y^n$ is in $R$ for all positive integer $n$. It is to be noted that in the Noetherian case all types of integrality coincide. In this talk we are going look at some of its "good" properties and give counterexamples to illustrate some of its "bad" properties.