MATH 166
MATH ADVENTURES CONTINUED

This problem is based on a question that I received from a former student in the summer of 2002. He was working as an engineer and needed to be able to find the following volume.

Consider the (3-dimensional) torus pictured below (one picture is the whole torus and the other is the torus "cut in half"). Assume that the radius of the inner circle of the torus is r (see the picture of the torus cut in half) and the the distance from the center of the torus to the center of the smaller circle is R ($R > r$). This inner tube (torus) is standing vertically (as in a tire swing) and has some water in it (say the height of the water as measured from the bottom of the torus is h). When my former student asked me the question intitially, the height of the water was less than the radius of the smaller circle, r, (and in any case $h \leq 2(r + R)$ which is the total height of the torus). But see if you can find a formula for the volume of the water in the torus for any value of h such that $0 \leq h \leq 2(r + R)$.