FLOW IN OPEN CHANNELS

\[ R_e = \frac{R_h V}{\nu} \]

Usually, \( R_h > 0.1 \) ft and \( V > 1 \) ft/s and \( \nu_{\text{water}} = 10^{-5} \) ft\(^2\)/s

Hence, \( R_e = 0.1 \times 1/10^{-5} = 10^4 \)

\[ R_e > 750 \]

\[ \therefore \text{FLOW IN OPEN CHANNELS IS ALMOST ALWAYS TURBULENT} \]

APPLY ENERGY EQUATION

\[ \frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + h_L \]

But,

\[ \frac{p_1}{\gamma} + z_1 = y_1 + S_0 \Delta X \quad \text{AND} \quad \frac{p_2}{\gamma} + z_2 = y_2 \]

\[ y_1 + S_0 \Delta X + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + h_L \]

If Channel bottom is Horizontal and \( h_L \) is neglected
\[ y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} \]

\[ y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} = \text{Constant!} \]

*depth of Channel + Velocity Head is Constant (If frictional losses=0 AND bed of channel is flat.)*

Let's call this constant: **Specific Energy**

\[ E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2} \]

We like to write flow rate per unit width of channel: so define \[ q = \frac{Q}{b} \]

\[ E = y + \frac{q^2}{2gy^2} \]

Or, \( (E – y)y^2 = \frac{q^2}{2g} \): Note a cubic equation with 3 roots. (third one is negative and has no physical meaning)

\( (E – y)y^2 = \frac{q^2}{2g} = \text{constant for a given} \ q \)

So if we plot \( E \) vs \( y \) that's a hyperbola with two values of \( E \) for each value of \( y \)

The above diagram is called a “Specific Energy Diagram”

Point C is a dividing point in the flow:
At depth below C large velocities are observed and at depth above C: smaller velocities result.

The velocity at C is called “critical velocity”
The depth at C is called “critical depth”

Characteristics for Critical Flow

\[ E = y + \frac{Q^2}{2gA^2} \]
\[ \frac{dE}{dy} = 0 \text{ and } dA = Tdy \]
\[ 0 = 1 + \left( \frac{Q^2}{gA^3} \right) \frac{dA}{dy} \]
\[ \frac{A^3}{T} = \frac{Q^2}{g} \]

\( y_c \) and \( V_c \) can be obtained as follows: (For rectangular channel)

\[ E = y + \frac{V^2}{2g} \]
Differentiate w.r.t \( y \)

or \( \frac{dE}{dy} = 0 \) at \( y = y_c \)

\[ 0 = 1 - \frac{q^2}{gy_c^3} \text{ or } q^2 = gy_c^3 \]

Since \( Q = VA = V(by) \) So, \( q = \frac{Q}{b} = Vy = V_c y_c \)

\( (V_c y_c)^2 = gy_c^3 \)

or \( y_c = \frac{V_c^2}{g} = (q^2/g)^{1/3} \) ....rectangular channel

Also the energy at the critical point is MINIMUM energy given by

\[ E_{\text{min}} = E_c = y_c + \frac{V_c^2}{2g} = 1.5y_c \text{ Also, } y_c = (2/3)E_{\text{min}} \]

Criterion for supercritical and subcritical flow

Critical flow at \( V_c = (gy_c)^{1/2} \)
Supercritical: \( V > (gy)^{1/2} \)

Subcritical: \( V < (gy)^{1/2} \)

Define a number “Froude Number”

\[ F = \frac{V}{(gy)^{1/2}} \]

So, \( F = 1 \): critical

\( F > 1 \): supercritical flow

\( F < 1 \): subcritical flow

What about slope of channel bed at critical flow?

Recall, manning's equation for flow rate:

\[ Q = \frac{1.449}{n} AR_{h}^{2/3} S_{0}^{1/2} \]

For wide and shallow rectangular channel: \( R_{h} = y \)

Critical slope is given by:

\[ S_{c} = \left( \frac{n}{1.486} \right)^{2} \frac{g}{y_{c}^{1/3}} \]

**CHARACTERISTICS OF CRITICAL, SUPER AND SUBCRITICAL FLOW**

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<th>SUPER</th>
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<td>Velocity</td>
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<td>( y = y_{c} = (q^{2}/g)^{1/3} )</td>
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<td>Mild slope</td>
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<td>Froude number</td>
<td>( F &gt; 1 )</td>
<td>( F = 1 )</td>
<td>( F &lt; 1 )</td>
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Since

\[ E = y + \frac{V^{2}}{2g} = y + \frac{Q^{2}}{2gA^{2}} = y + \frac{q^{2}}{2gy^{2}} \]

For a fixed specific energy, plot \( q \) vs \( y \)
At critical flow, discharge is maximum for a given specific energy.

EXAMPLES OF OCCURANCE OF CRITICAL FLOW

Physical sign of critical depth: “Wavy water surface”

- Sluice gate at broad crested weir

\[ q_{\text{max}} \]

\[ y_c \]

At a critical gate opening, gate has no effect on flow.

- Free outfall: effect of gravity
  Mild slope: curvature of outfall

\[ y_c \]

\[ S_0 < S_c \]

\[ 0.72 y_c \]

\[ 3 \text{ to } 10 y_c \]
Steep Slope

- Change in slope (break in grade) (sub to super)

- Change in elevation of channel bed

Supercritical

Subcritical
- Change in width of channel (changing $q$)

**Supercritical**

$y_2 > y_1$

**Subcritical**

$y_2 < y_1$