VISCOSITY

Resistance to motion in fluid.
Let's derive a mathematical description of viscosity:
Consider fluid to be made of layers. Consider motion of this fluid along a solid boundary. At the boundary fluid velocity is zero and at uppermost layer it is some finite velocity. (no slip condition) “velocity gradient” exists across distance y.

\[ \text{Strain} = \frac{(d_2 - d_1)}{dy} \]
Where displacement, \( d = \text{velocity} \times \text{time} \)

Hence, \( \text{strain} = \frac{dv}{dt} = \frac{dv}{dy} \)

And \( \text{strain rate} = \frac{dv}{dy} \cdot \frac{1}{dt} = \frac{dv}{dy} \)

For many fluid the sthear stress between layers, \( \tau \)

\[ \tau = \mu \frac{dv}{dy} \]
where $\mu$ is “coefficient of viscosity” or “viscosity”, “dymanic viscosity”, “absolute viscosity”
So, basis of viscosity is “fluid friction”
Note: if $\frac{dv}{dy} = 0$, shear stress $= 0$

In the fluid where does viscosity arise from?

1. Attraction between molecules (cohesion)

2. Molecules in one layer move to another layer constantly. So molecule from slow layer moving to fast layer slows the layer and vice versa: momentum exchange occurs between the layers

Notice: since $\tau = \mu \frac{dv}{dy}$
In solids, shear stress $\propto$ magnitude of deformation

In fluids, shear stress $\propto$ rate of deformation
Now let's plot shear stress vs \( \frac{dv}{dy} \) for different fluids

\[
\tau = \mu \frac{dv}{dy}
\]

Types of fluids depending on the shape of above plot

**Newtonian**: viscosity does not change with deformation
**Non newtonian**: slope is not a straight line

- **Shear thinning**: slope decreases with deformation ("fluid gets thinner with shear") pseudoplastic
- **Shear thickening**: slope increases with deformation ("fluid gets thicker with deformation") Dilatent

**Ideal plastic**: sustains stress before suffering plastic flow.
Applications of Non Newtonian:

Concrete flow in pumps
Polymer industry
Paints
Ceramics industry

Units of viscosity

Since $\tau=\mu \frac{dv}{dy}$, $\mu=\tau/(dv/dy)$

Units of m: Poise , $1 \text{ P} = 0.1 \text{ Ns/m}^2$
CentiPoise, $1 \text{ cP} = 0.01 \text{ P}$
Viscosity of water at 68.4 °F is 1 cP

Dimensions of $\mu$:
=dimensions of shear stress/dimensions of dv/dy

$$\frac{M L T^{-2} L^{-2}}{L T^{-1} L^{-1}} = \frac{M L^{-1} T^{-2}}{T^{-1}} = M L^{-1} T^{-1}$$

Define: Kinematic Viscosity: $\nu = \mu/\rho$

Unit of $\nu$ : $\text{ft}^2/\text{s}$ or $\text{m}^2/\text{s}$ commonly used: Stoke, $1 \text{ St} = 1 \text{cm}^2/\text{s}$, $1 \text{ cSt} = 0.01 \text{ St}$
Dimensions of $\nu$ : $L^2 T^{-1}$
Variation of Viscosity with temperature and Pressure

$\mu$ is independent of pressure, $\nu$ varies with pressure

Both $\mu$ and $\nu$ vary with temperature