Let $s(t)$, $v(t)$, and $a(t)$ denote the functions for distance, velocity, and acceleration at time $t$, respectively. Let $t = 0$ be at 2:45 a.m., so $t = 1/2$ is at 3:15 a.m. We’re given that $s(0) = 0$ miles, $v(0) = 0$ mph, and $s(1/2) = 160$ miles, and we’re given that $a(t) = A$ for some constant $A$. The goal is to find $v(1/2)$.

Recall

\[
\begin{align*}
v(t) &= s'(t) & \int v(t) \, dt &= s(t) + C \\
a(t) &= v'(t) & \int a(t) \, dt &= v(t) + C.
\end{align*}
\]

Hence,

\[
\int a(t) \, dt = \int A \, dt = At + C = v(t).
\]

Since $v(0) = 0$, $C = 0$, so $v(t) = At$, where $A$ needs to be determined. Note that after finding $A$, we can then evaluate $v(1/2)$ to get the answer. The problem says we need to use anti-derivatives at some point, so here it goes:

Note that $s(t)$ is the anti-derivative of $v(t)$, so by the FTC:

\[
\int_0^{1/2} v(t) \, dt = s(1/2) - s(0) = 160 \text{ miles},
\]

and since we also know $v(t) = At$,

\[
\int_0^{1/2} v(t) \, dt = \int_0^{1/2} At \, dt = \frac{A t^2}{2} \bigg|_{0}^{1/2} = \frac{A}{8} \text{ miles}.
\]

Setting these two quantities equal to one another gives $A/8 = 160$, so $A = 1280$ miles/hr$^2$. Thus, $v(t) = 1280t$, so $v(1/2) = 640$ miles/hr is our answer.

Check!

We have

\[
\begin{align*}
s(t) &= 640t^2 \\
v(t) &= 1280t \\
a(t) &= 1280.
\end{align*}
\]

Note that $v(t) = s'(t)$ and $a(t) = v'(t)$, and that $s(0) = 0$, $s(1/2) = 160$, and $v(0) = 0$. 