NCL Throughput Optimization

NCL systems can be optimized for speed by partitioning the combinational circuitry and inserting additional NCL registers and corresponding completion components. However, NCL circuits cannot be partitioned arbitrarily; they can only be divided at component boundaries in order to preserve delay-insensitivity. The average cycle time for an NCL system, $T_{DD}$, can be estimated as the worse-case stage delay of any stage in the pipeline, where the delay of one stage is equal to twice the sum of the stage’s worse-case combinational delay and completion delay, to account for both the DATA and NULL wavefronts. Algorithm 1 depicts this calculation for an N-stage pipeline, where $D_{combi}$ and $D_{compi}$ are stage $i$’s combinational and completion delays, respectively.

$$T_{DDmax} = 2 \times (D_{combi} + D_{compi})$$

for $(i = 2$ to $N)$ loop

$$T_{DDtemp} = 2 \times (D_{combi} + D_{compi})$$

$$T_{DDmax} = \text{MAX}(T_{DDtemp}, T_{DDmax})$$

end loop

Algorithm 1. NCL $T_{DD}$ estimation.

NCL pipelining can utilize either of two completion strategies: full-word or bit-wise completion. Full-word completion, as shown in Figure 1, requires that the acknowledge signals from each bit in register be conjoined together by the completion component, whose single-bit output is connected to all request lines of register $i-1$. On the other hand, bit-wise completion, as shown in Figure 2, only sends the completion signal from bit $b$ in register $i$ back to the bits in register $i-1$ that took part in the calculation of bit $b$. This method may therefore require fewer logic levels than that of full-word completion, thus increasing throughput. In this example, bit-wise completion is faster (i.e., 1 gate delay vs. 2 gate delays), but it requires more area (i.e., 4 gates vs. 2 gates).

![Figure 1. Full-word completion.](image-url)
To maximize throughput while minimizing latency and area, the following algorithm should be used to optimally partition an NCL circuit. Steps 1 and 2 initially partition an NCL circuit into stages of primary components, where a primary component is defined as a component whose inputs only consist of the circuit’s inputs or outputs of components that have already been added to a previous stage. Steps 3 and 4 then calculate the combinational delay (i.e., $D_{comb}$) and completion delay (i.e., $D_{comp}$) for each stage and the maximum delay for the entire pipeline (i.e., $max\_delay$), utilizing both full-word and bit-wise completion strategies. Finally, Step 5 merges stages to reduce latency and area, as long as doing so does not decrease throughput. Note that when merging stages the new merged combinational delay (i.e., $merged\_comb$) is not necessarily $D_{comb_i} + D_{comb_{i+1}}$. Take for example two full adders in a ripple-carry adder: $D_{comb_i} = 2$ and $D_{comb_{i+1}} = 2$, but $merged\_comb = 3$, since the carry output of a full adder has only 1 gate delay.

1) $i = 1$
2) loop until all components are part of a stage
   add all primary components to stage $i$
   $i = i + 1$
end loop
3) $N = i - 1$
   $max\_delay_{FW} = 0$
   $max\_delay_{BW} = 0$
4) for $j$ in 1 to $N$ loop
   $D_{comb} = max$ delay of stage $i$’s components
   $B = # of outputs from stage $j$
   $D_{comp_j} = \lceil \log_4 B \rceil$
   if $((D_{comb} + D_{comp_j}) > max\_delay_{FW})$ then
     $max\_delay_{FW} = D_{comb} + D_{comp_j}$
   end if
   $B = # of inputs to stage $j$
   $max\_outputs = 1$
   for $i$ in 1 to $B$ loop
     $num\_outputs = number$ of outputs of stage $i$ generated by input $i$
     if $((num\_outputs > max\_outputs))$ then
       $max\_outputs = num\_outputs$
     end if
   end loop
end loop

Figure 2. Bit-wise completion.
Dcomp = \lceil \log_4 \text{max\_outputs} \rceil \\
if ((Dcomb + Dcomp) > \text{max\_delay\_BW}) then \\
\text{max\_delay\_BW} = Dcomb + Dcomp \\
end if \\
end loop \\
5) if (\text{max\_delay\_FW} > \text{max\_delay\_BW}) then -- bit-wise design is faster \\
num\_stages = \text{call merge\_BW function} \\
output bit-wise pipelined design \\
e l s i f (\text{max\_delay\_BW} > \text{max\_delay\_FW}) then -- full-word design is faster \\
num\_stages = \text{call merge\_FW function} \\
output full-word pipelined design \\
e l s e \\
num\_stages\_BW = \text{call merge\_BW function} \\
num\_stages\_FW = \text{call merge\_FW function} \\
if (\text{num\_stages\_BW} > \text{num\_stages\_FW}) then -- full-word design has less latency \\
output full-word pipelined design \\
e l s i f (\text{num\_stages\_FW} > \text{num\_stages\_BW}) then -- bit-wise design has less latency \\
output bit-wise pipelined design \\
e l s i f (\text{area of full-word design} > \text{area of bit-wise design}) then -- bit-wise design is smaller \\
output bit-wise pipelined design \\
e l s e \\
output full-word pipelined design \\
end if \\
end if \\
\text{merge\_FW function} \\
um\_stages = N \\
for k in 1 to N-1 loop -- merge stages to decrease latency \\
merged\_comb = \text{max combinational delay of stage}_k \text{ and stage}_{k+1} \text{ merged into a single stage} \\
if (\text{merged\_comb} + \text{comp}_{k+1} \leq \text{max\_delay\_FW}) then \\
merge \text{stage}_k \text{ into stage}_{k+1} \\
delete \text{stage}_k \\
num\_stages = \text{num\_stages} - 1 \\
end if \\
end loop \\
return \text{num\_stages} \\
\text{merge\_BW function} \\
um\_stages = N \\
for k in 1 to N-1 loop -- merge stages to decrease latency \\
merged\_comb = \text{max combinational delay of stage}_k \text{ and stage}_{k+1} \text{ merged into a single stage} \\
B = \# \text{ of inputs to stage}_k \\
max\_outputs = 1 \\
for i in 1 to B loop \\
\text{num\_outputs} = \text{number of outputs of stage}_{k+1} \text{ generated by input}_i \\
if (\text{num\_outputs} > \text{max\_outputs}) then \\
\text{max\_outputs} = \text{num\_outputs} \\
end if \\
end loop \\
merged\_comp = \lceil \log_4 \text{max\_outputs} \rceil \\
if (\text{merged\_comb} + \text{merged\_comp} \leq \text{max\_delay\_BW}) then \\
merge \text{stage}_k \text{ into stage}_{k+1} \\
delete \text{stage}_k \\
num\_stages = \text{num\_stages} - 1 \\
end if \\
end loop \\
return \text{num\_stages} \\

\textbf{Algorithm 2. NCL pipelining algorithm.}
As an example, the non-pipelined quad-rail multiplier in Figure 3 has a worse-case combinational delay of 8 and a completion delay of 1, such that $T_{DD} = 18$. Applying Steps 1-4 of the pipelining algorithm to the quad-rail multiplier yields the results shown in Tables 1 and 2 for full-word and bit-wise completion, respectively. These tables show that the full-word pipelined design has a $T_{DD}$ (i.e. $2 \times \text{max\_delay}$, to account for both the DATA and NULL wavefronts) of 10 gate delays, while the bit-wise pipelined design has a $T_{DD}$ of 8 gate delays; hence the bit-wise pipelined design is preferred, since it maximizes throughput. Applying Step 5 of the algorithm to merge stages results in both the full-word and bit-wise pipelined designs merging Stages 3 and 4, such that both designs only require 3 stages. The new $D_{comb}$ is 3 and the new stage delay for both designs is 4. Note that $\text{max\_outputs}$ for the bit-wise design changes to 2 for the merged stage, such that $D_{comp}$ becomes 1.

**Table 1. Full-word completion pipelining.**

<table>
<thead>
<tr>
<th>Stage</th>
<th>$D_{comb}$</th>
<th># Outputs</th>
<th>$D_{comp}$</th>
<th>delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>8</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>6</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>5</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>max_delay</strong></td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stage</th>
<th>$D_{comb}$</th>
<th>$\text{max_outputs}$</th>
<th>$D_{comp}$</th>
<th>delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>max_delay</strong></td>
<td>4</td>
</tr>
</tbody>
</table>

**Figure 3. 4-bit × 4-bit unsigned quad-rail multiplier.**

**Table 2. Bit-wise completion pipelining.**

<table>
<thead>
<tr>
<th>Component</th>
<th>Output Gate Delays</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>Carry / PPH</td>
</tr>
<tr>
<td>Q33mul</td>
<td>1</td>
</tr>
<tr>
<td>Q332add</td>
<td>3</td>
</tr>
<tr>
<td>Q322add</td>
<td>2</td>
</tr>
<tr>
<td>Q32add</td>
<td>2</td>
</tr>
<tr>
<td>Q2Dadd</td>
<td>N/A</td>
</tr>
<tr>
<td>Q3Dadd</td>
<td>N/A</td>
</tr>
</tbody>
</table>
NCL system throughput can also be increased by applying the NULL Cycle Reduction (NCR) technique, depicted in Figure 4, which increases the throughput of an NCL system by decreasing the circuit’s NULL cycle time, without affecting its DATA cycle time. Successive input wavefronts are partitioned so that one circuit processes a DATA wavefront, while its duplicate processes a NULL wavefront. The first DATA/NUL cycle flows through the original circuit, while the next DATA/NUL cycle flows through the duplicate circuit. The outputs of the two circuits are then multiplexed to form a single output stream. NCR can be used to speedup slow stages in a NCL pipeline that cannot be further divided (e.g., Stage 2 in the quad-rail multiplier shown in Figure 3). The application of NCR to only the slow stages in a pipeline increases the throughput for the entire pipeline. NCR can also be used to increase the throughput of a feedback loop, which cannot be increased by any other means, again increasing throughput for the entire pipeline. Figure 4 depicts the NCR architecture for a dual-rail logic circuit utilizing full-word completion; however, NCR is also applicable to quad-rail circuits and bit-wise completion. Quad-rail logic only requires a redesign of the Demultiplexer and Multiplexer circuits to handle quad-rail signals, whereas bit-wise completion requires removal of the Completion Detection component and replication of the Sequencer components, such that each input/output bit has its own Sequencer#1/Sequencer#2 component, respectively.

Figure 4. NCR architecture for a dual-rail circuit utilizing full-word completion.