

SYLLABUS FOR NDSU ALGEBRA QUALIFYING EXAM

The following is a suggested list of topics for the algebra qualifying exam. It is not meant to be all-inclusive.

1. Groups

Subgroups, quotient groups, cosets, permutation groups, symmetric and alternating groups, homomorphism and isomorphism theorems, p -groups, group actions, Sylow subgroups, Abelian groups, solvable groups, nilpotent groups, Jordan-Hölder Theorem, Lagrange's Theorem, Cayley's Theorem, Sylow's Theorems, structure of finitely generated Abelian groups.

2. Rings

Ideals, quotient rings, polynomial rings, endomorphism rings, rings of fractions, localization and local rings, prime ideals, maximal ideals, integral domains, Euclidean rings, principal ideal domains, unique factorization domains, matrix rings, artinian rings, noetherian rings, Hilbert Basis Theorem, finite-dimensional algebras, Jacobson radical, semisimple rings, Jacobson Density Theorem, Artin-Wedderburn Theorem.

3. Modules

Submodules, quotient modules, free modules, projective modules, injective modules, Hom, tensor products, structure of finitely generated modules over principal ideal domains, exact sequences, vector spaces, vector space dimension, linear transformations, matrices, canonical forms, diagonalization, Cayley-Hamilton Theorem, complexes, homology, Ext, Tor.

4. Fields and Galois Theory

Extension fields, degrees, roots, geometric constructions, splitting fields, algebraic closure, algebraic and transcendental extensions, separable extensions, normal extensions, Galois extensions, Galois groups, fundamental theorem of Galois theory, solvability by radicals, constructibility, finite fields, primitive elements, cyclotomic extensions, cyclic extensions, radical extensions.

5. Category Theory

Categories, functors.

REFERENCES

1. David S. Dummit and Richard M. Foote, *Abstract algebra*, third ed., John Wiley & Sons Inc., Hoboken, NJ, 2004. MR 2286236 (2007h:00003)
2. Thomas W. Hungerford, *Algebra*, Graduate Texts in Mathematics, vol. 73, Springer-Verlag, New York, 1980, Reprint of the 1974 original. MR 600654 (82a:00006)
3. Serge Lang, *Algebra*, third ed., Graduate Texts in Mathematics, vol. 211, Springer-Verlag, New York, 2002. MR 1878556 (2003e:00003)

Date: September 11, 2008.