MATH 725, FALL 2012, HOMEWORK 7
DUE FRIDAY 07 DECEMBER

Exercise 1. Let $(A, \mathfrak{n})$ be a commutative local noetherian ring and consider the power series ring $R = A[\![X_1, \ldots, X_n]\!]$. Recall that R is commutative, local, and noetherian with maximal ideal $\mathfrak{m} = \mathfrak{n}R + (X_1, \ldots, X_n)R$. Prove that $\dim(R) = \dim(A) + n$.

Exercise 2. Let $(R, \mathfrak{m})$ be a commutative local noetherian ring, and let $\mathbf{x} = x_1, \ldots, x_n \in \mathfrak{m}$. Prove that if $\dim(R/(\mathbf{x})) = \dim(R) - n$, then $\mathbf{x}$ is part of a system of parameters for R.

Exercise 3. Let k be a field and set $R = k[\![X, Y, Z, W]\!]/(XY, ZW)$.

(a) Prove that $\dim(R) = 2$.

(b) Find a system of parameters for the ring $k[\![X, Y, Z, W]\!]/(XY - ZW)$.