Quiz Set 1
For Quiz on Thursday, January 30

Work all of the following problems. A subset of the problems and definitions from Chapter 2 will be on Quiz 1 to be given January 30. Quizzes will be graded for correctness and clarity.

(1) (Gallian, Chapter 2 Exercises, #14) For group elements a, b, and c, express $(ab)^3$ and $(ab^{-2}c)^{-2}$ without parentheses.

(2) (Gallian, Chapter 2 Exercises, #16) Show that the set $G = \{5, 15, 25, 35\}$ is a group under multiplication modulo 40.

(3) (Gallian, Chapter 2 Exercises, #26) Prove that in a group, $(a^{-1})^{-1} = a$ for all a.

(4) (Gallian, Chapter 2 Exercises, #27) For any elements a and b from a group and any integer n, prove that $(a^{-1}ba)^n = a^{-1}b^n a$. *Note:* You need to consider three cases: $n < 0, n = 0$ and $n > 0$.

(5) (Gallian, Chapter 2 Exercises, #33) Suppose the table below is a group (i.e., Cayley) table. Fill in the blank entries.

	e	a	b	c	d
e	e				
a		b			e
b		c	d	e	
c		d		a	b
d					

(6) (Gallian, Chapter 2 Exercises, #34) Prove that in a group, $(ab)^2 = a^2b^2$ if and only if $ab = ba$.

(7) (Gallian, Chapter 2 Exercises, #38) Give an example of a group with elements a, b, c, d, and x such that $axb = cxd$ but $ab \neq cd$. (Hence, "middle cancellation" is not valid in groups.)

(8) (Gallian, Chapter 3 Exercises, #6) In the group $\mathbb{Z}_{12}$, find $|a|$, $|b|$ and $|a + b|$ for each case.

(a) $a = 6, b = 2$

(b) $a = 3, b = 8$

(c) $a = 5, b = 4$

Do you see any relationship between $|a|$, $|b|$ and $|a + b|$?

(9) (Gallian, Chapter 3 Exercises, #49) Suppose a group contains elements a and b such that $|a| = 4, |b| = 2$ and $a^3b = ba$. Find $|ab|$.

(10) Let G be a group and let $g \in G$. Define a function $\phi_g : G \to G$ by $\phi_g(x) = gxg^{-1}$, where g^{-1} is the inverse of g, for all $x \in G$. Show that ϕ_g is one-to-one and onto. (Recall that a function f is *one-to-one* if whenever $f(a) = f(b)$ we must have $a = b$. Recall that a function $f : S \to T$ is *onto* if for each $t \in T$ there is an element $s \in S$ such that $f(s) = t$.)

(11) Let G be a group. For elements $a, b \in G$, define $a \sim b$ if there is some $x \in G$ with $a = xbx^{-1}$. Prove that $\sim$ is an equivalence relation on G.