Determination of ponding condition and infiltration into layered soils under unsteady rainfall

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Abstract

Few infiltration models, based on the Green–Ampt approach, in current use are suitable for simulating infiltration into heterogeneous soils of variable initial moisture distributions during unsteady rainfall. An algorithm is proposed for determining the ponding condition, simulating infiltration into a layered soil profile of arbitrary initial water distributions under unsteady rainfall, and partitioning the rainfall input into infiltration and surface runoff. Two distinct periods, pre-ponding and post-ponding, are taken into account. The model tracks the movement of the wetting front along the soil profile, checks the ponding status, and in particular, handles the shift between ponding and non-ponding conditions. The modeling also covers a fully saturated flow condition when the wetting front reaches the bottom of the soil profile (water table). Detailed procedures that deal with the variability in both soil properties and rainfall intensity are presented. For the purpose of model testing, three different cases (homogeneous soil and unsteady rainfall; layered soils and steady rainfall; and layered soils and unsteady rainfall) were discussed. Comparisons of the developed model with other infiltration models (both modified Green–Ampt infiltration model and fully numerical model) and field measurements were conducted and good agreements were achieved.

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1. Introduction

Watershed hydrologic modeling requires partitioning of surface runoff and infiltration for a rainfall event. Various modified Green–Ampt models have been widely used for this purpose. Mein and Larson (1973) developed a two-stage model that simulated infiltration into a homogeneous soil under steady rainfall. Two distinct infiltration stages, before and after surface ponding, were considered. Their infiltration model has been incorporated in the latest version of the SWAT package (Neitsch et al., 2002). Determination of ponding time and variation of infiltration capacity during steady rainfall was also discussed by Diskin and Nazimov (1996). Chu (1978) further extended the Green–Ampt approach to compute infiltration into a homogeneous soil under an unsteady rainfall event.

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In particular, a time shift method was proposed to deal with the integrated infiltration computations for stages with and without surface ponding. Also, specific implementation procedures were described in detail. Generally, these types of modified Green–Ampt infiltration models, including the corresponding components in some large, comprehensive modeling systems, such as SWAT (Neitsch et al., 2002) and HEC-HMS (USACE, 2001), are only applicable to homogeneous soils.

In the effort of modeling infiltration into nonuniform soils, Childs and Bybordi (1969) and Bybordi (1973) extended the Green–Ampt approach to model infiltration into a layered soil profile with hydraulic conductivity decreasing from the surface. As in the original Green and Ampt equation, an initially ponded condition was assumed in their derivations. Hachum and Alfaro (1980) expressed their modified version of the Green and Ampt equation for vertical infiltration into a nonuniform soil profile by using a harmonic mean of effective hydraulic conductivities of wetted soil layers. A steady rainfall application was assumed in their discussion. Similarly, Beven (1984) and Selker et al. (1999) also presented extensions of the Green and Ampt model for infiltration into soils of variable pore size with depth. Their discussions also were based on the assumption of an initially ponded condition.

However, few of the infiltration models, based on the Green–Ampt approach, in current use are suitable for modeling infiltration into a heterogeneous soil profile under an unsteady rainfall event, which is often the case in field conditions. Infiltration into a vertically nonuniform soil profile is strongly affected by the variability in soil properties. Meanwhile, unsteady rainfall makes the infiltration process more complicated due to the recurrence of ponding and non-ponding conditions. Thus, an algorithm that depicts the computation of infiltration of unsteady rainfall into a layered soil profile will be useful for practical applications. In this paper, we examine how to determine the ponding status for a layered soil profile under unsteady rainfall, how to simulate the subsequent infiltration and rainfall excess during the post-ponding period, and in particular, how to deal with the shift between ponding and non-ponding computations. Additionally, our model is tested for several cases and compared with other existing models.

2. Analysis and model development

2.1. Infiltration into layered soils vs. Green–Ampt approach

The infiltration process can be complicated due to the variability in soil texture. Hydraulic properties of different soil layers and their arrangement/combination may have significant influence on infiltration. For water movement from coarse to fine soils, when the wetting front first reaches the finer soil, infiltration rates may be slightly increased as a thin layer is wetted due to the greater attraction for water of the underlying finer soil (the suction head of the finer soil is generally higher) and then resistance to flow owing to the fineness of the pores may be so great that flow is markedly reduced (Miller and Gardner, 1962). Resultantly, due to the significant reduction in infiltration, a marked plateau and a step change in infiltration rates can be observed at the interface (Miller and Gardner, 1962; Bruce and Whisler, 1973). The Green–Ampt approach has been effectively applied to layered soils of decreasing permeability (e.g. Childs and Bybordi, 1969). In contrast, when water moves into a coarser soil, water cannot enter the coarse-textured soil layer until the pressure has built up sufficiently to wet the larger pores and, in particular, narrow flow channels (fingers) may occur and persist through the entire coarse-textured zone (Jury and Horton, 2004). In this circumstance, the underlying coarse soil layer will not be saturated. Thus, a commonly-used Green–Ampt model, in which a sharply-defined wetting front and piston flow are assumed, cannot be used for this case. For layered soils of increasing permeability such as crusted soils, Ahuja (1983) developed a Green–Ampt-type approach to deal with infiltration into crusted soils.

When the wetting front passes through the interface of two distinct textured soil layers, the abrupt change in soil hydraulic properties will result in a certain adjustment of water flow conditions in order to reach a new internal equilibrium. Additionally, the actual wetting front may not be a sharply-defined one and hence the adjustment process may start a little bit earlier. As a result, the transition from high to low infiltration rates can be smoother within a small neighborhood of the interface during the short adjustment time period. In a multi-layer Green–Ampt based model, however, the short adjustment across
the interface is neglected and an instantaneous hydraulic equilibrium at the interface is often assumed, which is consistent with the basic assumption of piston flow.

2.2. Modified Green–Ampt model for computing infiltration into layered soils

The soil profile under consideration consists of \( N_c \) soil layers with saturated hydraulic conductivities \( K_1, K_2, \ldots, K_{N_c} \). Infiltration into the layered soil profile under an initially ponded condition is schematically shown in Fig. 1 (assuming that the depth of ponded water on the surface is negligible). By introducing the assumption of instantaneous hydraulic equilibrium at each interface between distinct soil layers, a modified Green–Ampt model for simulating infiltration into this nonuniform soil profile is developed as follows.

2.2.1. Wetting front in layer 1

When the wetting front reaches a depth of \( z \) within the first layer \((0 < z \leq z_1)\), the infiltration rate \( i_z \) can be expressed in the form of Darcy’s law:

\[
i_z = V_1 = -K_1 \frac{dH}{dz} = \frac{K_1}{z - z_0} (H_0 - H_z) = \frac{K_1}{z} (z + h_{s1})
\]  

(1)

where \( V_1 \) is the percolation velocity of soil water in layer 1 [L/T]; \( K_1 \), the effective hydraulic conductivity of layer 1 [L/T]; \( H_0 \), the hydraulic head at \( z_0 \) [L] (note \( z_0 = 0 \) and \( H_0 = 0 \)); \( H_z \), the hydraulic head at \( z \) (wetting front) [L]; \( h_{s1} \) is the pressure head (suction) in layer 1 [L]. Note that for simplification purpose, the time-varying depth of ponded water at the ground surface is commonly neglected in derivations of a Green–Ampt based model. That is, the hydraulic impact of the ponding depth on water flow (pressure head corresponding to the thin ponded water) is assumed negligible. This assumption is also used in this model. However, as shown in the following sections, the surface storage of the ponded water that varies in time and its effects are simulated in the model.

The cumulative infiltration \( I_z \) is given by

\[
I_z = \int_0^z i_z \, dz = \int_0^z \frac{K_1}{z} (z + h_{s1}) \, dz
\]  

(2)

where \( \theta_{s1} \) is the saturated water content of layer 1 [L^3/L^3]; \( \theta_{i1} \), the initial water content of layer 1 [L^3/L^3]; \( \theta_{i1} \) is the difference between saturated water content and initial water content of layer 1 [L^3/L^3].

Since \( i_z = \frac{dI_z}{dt} \),

\[
\int_0^{t_z} dt = \int_0^z \frac{\theta_{i1} z}{K_1(z + h_{s1})} \, dz
\]  

(3)

Hence, the time for the wetting front to arrive at location \( z \) can be written as

\[
t_z = \frac{\theta_{i1} z - \theta_{i1} h_{s1} \ln\left( \frac{z + h_{s1}}{h_{s1}} \right)}{K_1}
\]  

(4)

2.2.2. Wetting front in layer 2

When the wetting front reaches a depth of \( z \) within layer 2 \((z_1 < z \leq z_2)\), the uniform velocity of the wetting front can be expressed as

\[
i_z = V_1 = V_2
\]  

(5)

i.e.

\[
\frac{K_1}{z_1 - z_0} (H_0 - H_1) = \frac{K_2}{z - z_1} (H_1 - H_z)
\]  

(6)

Note that instantaneous hydraulic equilibrium is assumed when the wetting front passes through the interface of the distinct soil layers. Substitution of the upper and lower boundary conditions, \( H_0 = 0 \) and \( H_z = -z - h_{s2} \) into Eq. (6) gives

\[
H_1 = -\frac{K_1}{z_1} \left( \frac{z - z_0}{k_1} + \frac{z - z_1}{k_2} \right)
\]  

(7)
\[ i_c = \frac{z + h_{s2}}{\frac{z-z_0}{K_1} + \frac{z-z_1}{K_2}} \]  

(8)

To keep the continuity in pressure potential, an average hydraulic head can be used when the wetting front is located at the interface \( (H_1 = -z_i = (h_{s1} + h_{s2})/2) \) for \( z = z_1 \). The cumulative infiltration can be written as

\[ I_z = I_{z_1} + (z - z_1)(\theta_{s2} - \theta_{s1}) \]

(9)

\[ i_c = \frac{dI_c}{dt} = \theta_{s2} \frac{dz}{dt} \]

the travel time from \( z_1 \) to \( z \) in layer 2 is then given by

\[ t_c = t_{z_1} + \frac{\theta_{s2}}{K_2} (z - z_1) \]

(10)

or

\[ t_c = t_{z_1} + \frac{\theta_{s2}}{K_2} (z - z_1) \]

(11)

in which \( t_{z_1} \) can be obtained from Eq. (4) by replacing \( z \) with \( z_1 \).

2.2.3. Wetting front in layer \( n \)

Similarly, when the wetting front is in layer \( n \) at location \( z = z_{n-1} \), the infiltration rate and cumulative infiltration can be, respectively, expressed as

\[ i_c = \frac{z + h_{s_{n-1}}}{\sum_{j=1}^{n-1} \frac{z-j_{s_{n-1}}}{K_j} + \frac{z-z_{n-1}}{K_n}} \]  

(13)

and

\[ I_z = I_{z_{n-1}} + (z - z_{n-1})(\theta_{s_{n-1}} - \theta_{s_{n-1}}) \]

(14)

From \( (dI_c/dt) = \theta_{s_{n-1}}(dz/dt) \), the travel time from \( z_{n-1} \) to \( z \) in layer \( n \) can be expressed as

\[ \int_{t_{z_{n-1}}}^{t_z} dt = \int_{z_{n-1}}^{z} \theta_{s_{n-1}} \left( \frac{\sum_{j=1}^{n-1} \frac{z-j_{s_{n-1}}}{K_j} + \frac{z-z_{n-1}}{K_n}}{z + h_{s_n}} \right) \]  

(15)

or

\[ t_c = t_{z_{n-1}} + \frac{\theta_{s_{n-1}}}{K_n} (z - z_{n-1}) \]

(16)

In addition to the upper and lower boundary conditions of the wetted zone \( (H_0 = 0 \) and \( H_n = -z - h_{s_n}) \), when the wetting front is located at depth \( z \) in layer \( n \), the general expressions for hydraulic heads at interfaces can be written as

\[ H_1 = -\frac{z + h_{s_n}}{\frac{K_1}{z_1} \left( \frac{\sum_{j=1}^{n-1} \frac{z-j_{s_{n-1}}}{K_j} + \frac{z-z_{n-1}}{K_n}}{K_n} \right)} \]  

(17)

\[ H_i = \frac{(\alpha_{i-1} + \alpha_i)H_{i-1} - \alpha_{i-1}H_{i-2}}{\alpha_i} \]  

(18)

in which

\[ \alpha_i = \frac{K_i}{z_i - z_{i-1}} \]

(19)

2.3. Pre-ponding computation and determination of ponding status

The entire soil profile is spatially divided into \( N_c \) soil layers and the time domain is discretized into \( N_t \) time steps with an increment of \( \Delta t \). In addition to reflecting the vertical variability of soil types, the spatial discretization of the soil profile can also account for the changes in the distribution of the initial water content. To deal with the variability in the rainfall intensity in the pre-ponding computation, a potential filling depth of rainfall is first calculated for each time step based on the soil water deficit and the amount of rainfall (if the entire soil profile is filled at
a certain time step, the filling depth for all remaining time steps is equal to the maximum depth). For time step \( m \), the cumulative potential filling depth of rainfall \( z_{nm} \) that is located in layer \( n_m \) is given by

\[
\sum_{j=1}^{n_m-1} (z_j - z_{j-1}) \theta_{ij} + (z_{nm} - z_{n_m-1}) \theta_{in_m} = \sum_{k=1}^{m} r_k \Delta t
\]  

(20)

\[
z_{nm} = z_{n_m-1} + \frac{\sum_{k=1}^{m} r_k \Delta t - \sum_{j=1}^{n_m-1} (z_j - z_{j-1}) \theta_{ij}}{\theta_{in_m}}
\]  

(21)

where \( r_k \) is the rainfall intensity at time step \( k \) [L/T].

In such a manner, the soil profile is further divided into \( N_t \) rainfall zones (note that the rainfall zoning is only used for the pre-ponding computation). By combining the soil layers and rainfall zones, one ends up with a set of soil-rainfall cells (Fig. 2). Each cell possesses uniform soil properties and constant rainfall intensity. The computation is conducted from the ground surface down through the soil profile. The soil property, rainfall intensity, or both are altered whenever the wetting front passes the interface of two cells. Clearly, all rainwater infiltrates into the soil before ponding and the actual infiltration rate \( f_m \) at time step \( m \) is equal to the rainfall intensity \( r_m \), which is less than infiltration capacity \( i_c \), i.e.

\[
f_m = r_m < i_c
\]  

(22)

If a ponded condition is achieved \( (r_m = i_c) \) at a depth of \( z_p \) in soil layer \( n_p \) and rainfall zone \( m_p \) at time \( t_p \), we have

\[
r_{m_p} = i_c = \frac{z_p + h_{str}}{\sum_{j=1}^{n_p-1} (z_j - z_{j-1}) / K_j + z_p - z_{n_p-1}}
\]  

(23)

Solving for \( z_p \)

\[
z_p = \frac{K_{n_p} h_{str} + r_{m_p} z_{n_p-1} - r_{m_p} K_{n_p} \sum_{j=1}^{n_p-1} (z_j - z_{j-1}) / K_j}{r_{m_p} - K_{n_p}}
\]  

(24)

The cumulative infiltration water quantity at the ponding condition is given by

\[
I_p = \sum_{j=1}^{n_p-1} (z_j - z_{j-1}) \theta_{ij} + (z_p - z_{n_p-1}) \theta_{in_p}
\]  

(25)

At the ponding time \( t_p \) \( (m_p - 1) \Delta t \leq t_p \leq m_p \Delta t \), the cumulative rainfall is given by

\[
Q_r = \int_0^{t_p} r(t) \, dt
\]

\[
= \sum_{k=1}^{m_p-1} r_k \Delta t + r_{m_p} [t_p - (m_p - 1) \Delta t]
\]  

(26)

From \( Q_r = I_p \), it follows that

\[
t_p = \frac{1}{r_{m_p}} \left( I_p - \sum_{k=1}^{m_p-1} r_k \Delta t \right) + (m_p - 1) \Delta t
\]  

(27)

Following the time shift method proposed by Chu (1978), the equivalent time for the wetting front to reach a depth of \( z_p \) under an initially ponded condition can be calculated by Eq. (16)

\[
t_{pd} = t_{m_p-1} + \theta_{in_p} \left( \frac{z_p - z_{n_p-1}}{K_{n_p}} \right)
\]

\[
+ \theta_{in_p} \left[ \sum_{j=1}^{n_p-1} \frac{z_j}{K_j} \left( \frac{1}{K_j + 1} - \frac{1}{K_{n_p}} \right) - \frac{h_{str}}{K_{n_p}} \right]
\]

\[
\times \ln \left( \frac{z_p + h_{str}}{z_{n_p-1} + h_{str}} \right)
\]  

(28)

Thus, the initial time at which a ponded condition is assumed is given by

\[
t_0 = t_p - t_{pd}
\]  

(29)
2.4. Post-ponding computation

The post-ponding computation starts from the ponding time \( t_p \) and the corresponding depth \( z_p \). Because the rainfall intensity is greater than infiltration capacity after ponding, the excess rainfall will first fill the surface storage and then contribute to surface runoff. Two unique procedures, mass balance checking and ponding status checking, are implemented for the post-ponding computation. The overall mass balance equation for the surface zone at time step \( k \) can be written as (neglecting the evaporation/evapotranspiration during the rainfall period):

\[
P_k = I_k + S_k + R_k
\]  
(30)

where \( P_k \) is the cumulative rainfall at time step \( k \) [L]; \( I_k \), the cumulative infiltration at time step \( k \) [L]; \( S_k \), the surface storage at time step \( k \) [L]; \( R_k \) is the cumulative surface runoff at time step \( k \) [L]. Under ponding conditions, the actual infiltration rate \( f_k \) equals the infiltration capacity \( i_z \), expressed by Eq. (13). The cumulative infiltration and infiltration time when the wetting front reaches layer \( n \) at a depth of \( z \) can be calculated using Eqs. (14) and (16), respectively. Note that the shifted time for the infiltration computation is given by

\[
t_z = t - t_0 = t - t_p + t_{pd}
\]  
(31)

For each time step ranging from \( t_{k-1} \) to \( t_k \), the infiltration computation is implemented layer by layer under a constant rainfall. During the ponding period, the infiltration capability is always satisfied. Thus, whether the wetting front is able to pass through a deeper soil layer primarily depends on the soil water deficit and the remaining time in the current time step. Assuming that the wetting front is located at depth \( z_{k-1} \) within layer \( n_{k-1} \) at the beginning time \( t_{k-1} \), it reaches a depth of \( z_{tk} \) within layer \( n_k \) at the ending time \( t_k \) after passing through \( J_k \) soil layers (Fig. 3). Given the ending time \( t_k \), the final exact location of the wetting front, \( z_{tk} \), is determined by applying the Newton–Raphson method to Eq. (16), and the corresponding infiltration rate at depth \( z_{tk} \) can be calculated using Eq. (13). Under this circumstance, the increment of the cumulative infiltration in this time step can be expressed as

\[
\Delta I_k = (z_{tk-1} - z_{tk-1}) \theta_{tn-1} - \sum_{j=1}^{J_k} (z_{tn-1} + j - z_{tn-1} + j) \theta_{tn-1} + j
\]

\[
+ (z_{tk} - z_{tn-1} + j) \theta_{tn_k}
\]  
(32)

From the mass balance Eq. (30), the increment of surface runoff is given by

\[
\Delta R_k = \Delta P_k - \Delta S_k - \Delta I_k
\]

\[
= r_k \Delta t + (S_{k-1} - S_k) - \Delta I_k
\]  
(33)

Hence, the following analyses hold as long as the ponding condition is valid:

For \( r_k \Delta t \leq \Delta I_k + (S_{k-1} - S_k) \):

\[
S_k = S_{k-1} + (r_k \Delta t - \Delta I_k)
\]  
(34)

\[
\Delta R_k = 0
\]  
(35)

where \( S_{max} \) is the surface storage capacity. Clearly, the surface storage will increase if \( r_k \Delta t > \Delta I_k \). Otherwise, a portion of the surface storage will be used to satisfy the infiltration capacity.

For \( r_k \Delta t > \Delta I_k + (S_{max} - S_{k-1}) \):

\[
S_k = S_{max}
\]

\[
\Delta R_k = r_k \Delta t - \Delta I_k - (S_{max} - S_{k-1})
\]  
(37)

The ponding status is checked for all time steps. If \( r_k < f_k = i_z \) and \( S_{k-1} = 0 \), the ponding condition is no longer satisfied and the computation has to be shifted back to the pre-ponding computation. However, before a new pre-ponding modeling, the rainfall zoning needs
to be updated because some rainfall in previous time steps may have contributed to surface runoff, instead of completely infiltrating into the soil profile. Hence, the actual simulated filling depths are used for all previous time steps. Starting from the current location of the wetting front, new filling depths are computed for the remaining rainfall time steps. Consequently, a new soil-rainfall cell system is generated. Then, a new cycle of infiltration modeling starts.

2.5. Fully saturated flow computation

For both pre-ponding and post-ponding infiltration computations, if the wetting front reaches the bottom of the entire soil profile, a fully saturated condition is achieved. In this case, the infiltration capacity is assumed constant and the capillary head is assumed zero beyond the bottom of the soil profile (water table). It is also assumed that water flows vertically in the simulated soil profile and horizontally in the underlying aquifer and there is no increase in the water table. Thus, Eq. (13) for the infiltration capacity is changed as:

$$i_N = \frac{z_N}{\sum_{j=1}^{N_s} \frac{z_j}{K_j}}$$

(38)

The fully saturated computations for the remaining time steps are also characterized by ponding and non-ponding conditions.

For the ponding condition, i.e. $(r_k + S_{k-1}/\Delta t) \geq i_{N_k}$, the actual infiltration rate and the increment of the cumulative infiltration can be, respectively, expressed as:

$$f_k = i_{N_k}$$

(39)

$$\Delta I_k = f_k \Delta t$$

(40)

The surface storage and the increment of the surface runoff can be calculated using Eqs. (34)–(37) for different situations.

If $(r_k + S_{k-1}/\Delta t) < i_{N_k}$ (non-ponding condition), all available water infiltrates into the soil profile and the actual infiltration rate is given by

$$f_k = r_k + S_{k-1}/\Delta t$$

(41)

This is also the actual drainage rate at the bottom for the time step. In this circumstance, the surface storage will be zero by the end of the time step $k$ and no water contributes to the surface runoff, i.e.

$$S_k = 0$$

(42)

$$\Delta R_k = 0$$

(43)

3. Model testing

The model, developed herein based on the Green–Ampt approach, is suitable for simulating infiltration into a layered soil profile (including one-layer homogeneous soils) with arbitrary distributions of initial soil water content under either steady or unsteady rainfall events. The model tracks the movement of the wetting front along the soil profile, checks the ponding status and mass balance, handles the shift between ponding and non-ponding conditions, and partitions variable rainfall inputs into infiltration and surface runoff. To test the performance of the model, three cases that cover different combinations of soil properties and rainfall features are examined in this section. For the case of homogeneous soil and unsteady rainfall (Case 1), a modified Green–Ampt infiltration model, developed by Chu (1978), is selected for comparison purposes. Subsequently, our discussion is further extended to layered soils under a steady water application (Case 2). Bruce and Whisler (1973) presented extensive field experimental studies on infiltration of water into layered soils. They also developed a numerical model that directly solved Richards’ equation for simulating infiltration and compared their modeling results with the field measurements. In this test case, we apply our model to one of their experiment plots (plot 6) and compare our modeling results with their simulated values, as well as their field measurements. Finally, a mixed case (Case 3) is examined for modeling infiltration into nonuniform soils under an unsteady rainfall event. In this case, the same soil profile that is used in Case 2 (Bruce–Whisler case) is selected. However, instead of a constant water application, an unsteady rainfall event based on Case 1 (Chu case) is used. These three test cases are presented in detail next.

3.1. Case 1: homogeneous soil and unsteady rainfall

As described in the field evaluation of Chu’s model (1978), a homogeneous soil and uniform distribution
of antecedent soil moisture are assumed. Average hydraulic parameters of silt loam and sandy loam, listed in Table 1, are adopted for this test case. Given the values of \( q_s \) and \( h_s \), the initial water content \( q_0 \) is determined so that \((q_s K q_0 h_s)\) = 3.6 cm, used in Chu’s original computation. The unsteady rainfall event (September 9, 1959) is selected for this test case. The time domain is discretized into 1333 time steps of an interval of 0.001 h. The modeling indicates that ponding condition is achieved at time \( t_p \) = 0.174 h (\( t_{pd} = 0.073 \) h; \( t_0 = 0.101 \) h) (Fig. 4a) when the wetting front moves to a depth \( z_p \) = 3.581 cm. At that point, the infiltration rate is equal to the rainfall intensity (6.91 cm/h; point A in Fig. 4a) and the corresponding cumulative infiltration is 0.931 cm. Comparisons of the simulated cumulative infiltration and runoff, and infiltration rate at different times between this model and Chu’s model are shown in Table 2 and Fig. 4 a and b. All values resulting from the two models match each other. Fig. 4a also indicates that the infiltration modeling shifts from the ponding condition back to a non-ponding condition at \( t \) = 1.084 h (point B) when the rainfall intensity (1.53 cm/h) is less than the infiltration capacity (2.546 cm/h). Such a non-ponding condition is held until the end of the entire simulation period. At time \( t = 1.333 \) h, the wetting front finally reaches a depth of 17.465 cm.

### 3.2. Case 2: layered soils and steady rainfall

Bruce and Whisler (1973) conducted a series of field experiments and numerically modeled infiltration into layered soil profiles. We apply our model to one of their experimental plots (plot 6) and compare our results with their numerical model and also with their field measurements. The entire soil profile is 120 cm and the modeling covers four distinct soil layers (A, B, B, and B) that are further divided into 120 cells with an increment of 1 cm. The soil types, distributions, and relevant hydraulic parameters are shown in Table 3. Water is applied to the plot at a constant rate of 12.6 cm/h with a duration of 2.0 h. In the modeling, the simulation time period of 2 h is discretized into 120 time steps with an interval of 0.016667 h (1 min). The surface storage capacity is assumed zero. As shown in Fig. 5, the initial soil moisture distribution used in the model varies with depth. Fig. 6a and b show the simulated cumulative infiltration and infiltration rate as a function of time, as well as comparisons with the values measured in the field and simulated by the Bruce and Whisler’s numerical model. Good agreement with the field measurements is observed. The graphs also illustrate the shifting of the wetting front from the ponding to non-ponding condition at different times, as well as the comparison of the simulated cumulative infiltration and runoff with the field measurements. The resulting graphs and comparisons highlight the effectiveness of our model in simulating infiltration and runoff processes in layered soil profiles.
measurements can be observed for both cumulative infiltration (Fig. 6a) and infiltration rate (Fig. 6b). The comparison with the field measurements indicates that the cumulative infiltration and the infiltration rate simulated by the current model are even better than those from the Bruce and Whisler numerical model. The ponding status is established when the rainfall intensity is equal to the infiltration capacity $r = i_{w} = 12.6$ cm/h at time $t_p = 0.458$ h ($t_{pd} = 0.255$ h; $t_0 = 0.203$ h). At that time, the wetting front reaches a depth of 23.167 cm in the soil layer B1 and thereafter, surface runoff starts. A plateau corresponding to the non-ponding condition (from 0 to 0.458 h) can be observed in the infiltration curve (Fig. 6b). Thereafter, infiltration rates, following the infiltration capacity curve, decrease until the wetting front reaches the interface between the soil layers B1 and B2 at a depth of 45 cm at time $t = 0.905$ h. As the wetting front moves into the finer textured soil layer B2 of a value of hydraulic conductivity that is about nine times less than that of the upper layer B1, a significant decrease in infiltration rates, primarily induced by the lower permeability of the layer B2, can be observed across the interface. Consequently, a plateau that coincides with the entry of the wetting front into the finer layer B2 is exhibited in the simulated infiltration rate curve at time $t = 0.905$ h. Bruce and Whisler (1973) also emphasized this marked plateau and step occurring in their simulated infiltration rate curve (Fig. 6b) (their simulated time to the interface is 1.17 h). As pointed out in their discussion, this phenomenon had been previously observed in layered soil columns by Miller and Gardner (1962). After carefully analyzing the measured infiltration rate data, we found the following features (Fig. 6c): (1) the observed data before 0.8 h follow a nice decreasing pattern and a good curve can be fitted; (2) a clear decreasing trend can also be observed.
observed for the observed data between 0.8 and 1.2 h; (3) a significant decrease in infiltration rates can be observed around the times between 0.8 and 0.9 h and the difference in infiltration rates can be as high as 4 cm/h. This significant change in infiltration rates coincides with the time when the wetting front passed the interface; and (4) the observed data for \( t > 1.2 \) h are considerably scattered. Therefore, the field measurements seem also to show a similar changing pattern in infiltration rates and overall agreement is satisfactory although a precise comparison of the simulated and measured infiltration rates is difficult (Bruce and Whisler, 1973). In addition, it can also be found from the observed data that the influence of the finer layer \( B_2 \) seems to start at \( t = 0.8 \) h that is earlier than the times simulated by this model (0.905 h) and Bruce and Whisler’s model (1.17 h). By the end of the simulation period \( (t = 2.0 \) h), the wetting front reaches a depth of 67.75 cm (still in layer \( B_2 \)) and the corresponding infiltration rate and cumulative infiltration are 1.525 cm/h and 12.312 cm, respectively. Comparisons of the water content distributions along the soil profile, simulated by this model and the Bruce and Whisler’s numerical model for four different times, are shown in Fig. 7a–d, respectively (note that there is a very small difference in the times used in the B–W model and this model). Good agreement in the timing and location of the wetting front simulated by the two models can be observed. Due to lack of observed data, we cannot compare the simulated soil moisture distributions with the observed ones for all four selected times. However, the locations of the wetting front at \( t = 2.0 \) h simulated by both models match the observed one (Fig. 7d).

### 3.3. Case 3: layered soils and unsteady rainfall

The purpose of this test case is to examine the effects of soil properties and rainfall on infiltration when both factors vary simultaneously. This is a mixed case, in which the soil profile and parameters used in Case 2 are applied. The unsteady rainfall event in Case 1 is used, but the rainfall intensity before \( t = 0.583 \) h is doubled so as to come up with a complex case that covers both ponding and non-ponding conditions, as well as a series of shifts between them (note that the ponding status cannot be achieved within the simulation period if using the original rainfall intensities). As in Case 1, the time domain is also discretized into 1333 time steps with an increment of 0.001 h and the soil profile consists of 120 cells with an increment of 1 cm. The surface storage capacity is assumed 0.3 cm. The rainfall intensity, simulated infiltration rate, and infiltration capacity are schematically shown in Fig. 8a. The cumulative rainfall, simulated cumulative infiltration and runoff, as well as the surface storage changes in time are depicted in Fig. 8b. It can be observed from
Fig. 8a and b that the entire simulation period is characterized by three non-ponding and two ponding sub-periods (four times of shifting between the two conditions). The first ponding status is achieved at time $t_p = 0.333$ h when the rainfall intensity suddenly increases to 19.92 cm/h, which is greater than the infiltration capacity, 19.346 cm/h (point A in Fig. 8a). At that time, the wetting front is located at a depth of 15.877 cm. Then, under the ponding condition, the rainfall excess starts to fill the surface storage but surface runoff begins until time $t = 0.43$ h when the surface storage reaches the capacity (0.3 cm) (Fig. 8b). At time $t = 0.584$ h, the rainfall intensity reduces from 19.92 to 4.93 cm/h, which is less than the corresponding infiltration capacity, 9.009 cm/h (point B in Fig. 8a). However, the ponding status still holds till all of the surface storage water infiltrates into the soils at time $t = 0.667$ h (point C in Fig. 8a). Then the ponding condition is shifted back to a non-ponding condition and another cycle begins. When the wetting front passes through the interface between the soil layers B$_1$ and B$_2$, the soil infiltration capacities

Fig. 8. Simulation results for Case 3 (nonuniform soils and unsteady rainfall).
significant decrease due to the huge reduction in permeability of the finer textured soil layer B₂, although a momentary slight increase in the infiltration capacity, primarily resulted from the higher suction head of the finer soil layer B₂, can be observed across the interface. This phenomenon, as described in Section 2.1, has also been observed by Miller and Gardner (1962).

The second ponding status is established at time \( t_p = 1.058 \) h (point D in Fig. 8a) when the wetting front moves down to a depth of 47.219 cm. This new ponding condition lasts only 0.029 h, during which the limited rainfall excess is available only for partially filling the surface storage and thus there is no water contribution to surface runoff (Fig. 8b). At time \( t = 1.084 \) h (point E in Fig. 8a), the rainfall intensity decreases to 1.53 cm/h and the water in the surface storage then begins to infiltrate into the soils. When the surface storage is zero at time \( t = 1.087 \) (point F in Fig. 8a), the ponding condition is again shifted back to a non-ponding condition and it holds till the end of the simulation (\( t = 1.333 \) h, point G in Fig. 8a). The wetting front is finally located at a depth of 50.973 cm (in layer B₂).

4. Summary and conclusions

An infiltration model was developed in this study based on the Green–Ampt approach. The model was suitable for simulating infiltration into nonuniform/uniform soils of arbitrary initial moisture distributions during an unsteady/steady rainfall event and able to deal with ponding and non-ponding conditions, and shifting between the two conditions. Furthermore, the model was tested for three different cases that represented various combinations of soil properties and rainfall features. In particular, the model was compared with some field measurements and other existing models, including a modified Green–Ampt infiltration model (Chu, 1978) and an unsaturated numerical model (Bruce and Whisler, 1973), and good agreements were achieved. Thus, it can be concluded that the developed model was able to provide satisfactory simulations of the infiltration processes that were strongly influenced by the vertical variability in soil hydraulic properties and complex changes in rainfall intensities. The model eliminated several common limitations in existing infiltration models based on the Green–Ampt approach, such as homogeneous soil, uniform initial water content, and initially ponded condition requirement and provided a generalized algorithm for determination of ponding condition and infiltration simulation. It should be pointed out that in the analyses and mathematical derivations, evaporation/evapotranspiration and soil water redistribution during the rainfall event were considered as not significant. Such processes will be taken into account in our further studies on infiltration into layered soils under complex rainfall patterns that include a series of wet (with rainfall) and dry (without rainfall) time periods.

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References


