Talent Search

The Department of Mathematics at NDSU is happy to announce the start of the annual North Dakota Mathematics Talent Search. The goals of the talent search are to locate high school students in North Dakota and surrounding areas with a talent for solving mathematical problems, to reward these students and their teachers for their efforts, and to encourage these students to attend NDSU and major in the mathematical sciences or engineering.

The Talent Search poses sets of challenging mathematical problems throughout the year which will be posted on our website at

https://www.ndsu.edu/math/ongoing_events/nd_talent_search/

Interested students are strongly encouraged to send in solutions even if they only solve one problem in a set; **finding a good solution to a problem is always an achievement**. The problems do not require advanced mathematical knowledge – just creativity and a feeling or taste for problem solving.

The students who submit a significant number of mathematically sound solutions for each of the three rounds will be rewarded with various prizes.

Please upload and submit your solutions by October 27, 2017, using the form on the website. Alternatively, solutions may be sent by regular mail to:

Talent Search
c/o Maria Alfonseca
Mathematics NDSU Dept.# 2750
PO BOX 6050
Fargo, ND 58108-6050

Please do not forget to include your name, postal address, school, and e-mail address.

Here is the first set of problems:

1. Find all the integer values of n such that $\frac{n+5}{n-4}$ is an integer.

2. A perfect number is defined as a number that equals the sum of its proper divisors. Example: The proper divisors of 6 are 1, 2, 3, and 1+2+3=6. You can also check that 28 is a perfect number.

   Show that the number $N = 2^k(1 + 2 + 4 + 8 + 16 + \cdots + 2^k)$ is perfect if we assume that $(1 + 2 + 4 + 8 + 16 + \cdots + 2^k)$ is a prime number.

3. There are seventeen points on a plane. Each pair of points is connected by a line segment of one of three colors: red, yellow and blue. Show that there are three points that are vertices of a triangle whose sides have the same color.

4. Everybody in town A always tells the truth. Everybody in town B always lies. In town C, people will alternate between telling the truth and lying. You are in one of these cities but don't know which one. How can you decide, by asking exactly four questions to a person in the street, in which of the cities you are?

5. We define a "distance" between points on the plane as follows: The distance between the points (a, b) and (c, d) is equal to the maximum of the two quantities $|c-a|$, $|d-b|$. For example, the distance between the points $(0, 0)$ and $(1, 1,)$ is 1.

   (a) Find all the points at distance 1 from $(0, 0)$. Draw the set of these points.

   (b) We define that a direction $\langle u, v\rangle$ is "perpendicular" to another direction $\langle w, z\rangle$ if, given any point (a, b) on the line passing through the point (u, v) and having direction $\langle w, z\rangle$, we have that the distance from (u, v) to $(0, 0)$ is always less than or equal to the distance from (a, b) to $(0, 0)$. Here we are using the new distance we defined at the beginning of the problem. A line is defined in the usual way.

   Show that the direction $\langle 1/2, 1\rangle$ is perpendicular to the horizontal direction, $\langle 1, 0\rangle$, but that the direction $\langle 1, 0\rangle$ is not perpendicular to $\langle 1/2, 1\rangle$.