Talent Search

The Department of Mathematics at NDSU is happy to announce the start of the annual North Dakota Mathematics Talent Search. The goals of the talent search are to locate high school students in North Dakota and surrounding areas with a talent for solving mathematical problems, to reward these students and their teachers for their efforts, and to encourage these students to attend NDSU and major in the mathematical sciences or engineering.

The Talent Search poses sets of challenging mathematical problems throughout the year which will be posted on our website at
https://www.ndsu.edu/math/ongoing_events/nd_talent_search/

Interested students are strongly encouraged to send in solutions even if they only solve one problem in a set; **finding a good solution to a problem is always an achievement**. The problems do not require advanced mathematical knowledge – just creativity and a feeling or taste for problem solving.

The students who submit a significant number of mathematically sound solutions for each of the three rounds will be rewarded with various prizes.

Please upload and submit your solutions by October 31, 2018, using the form on the website. Alternatively, solutions may be sent by regular mail to:

Talent Search
c/o Maria Alfonseca
Mathematics NDSU Dept.# 2750
PO BOX 6050
Fargo, ND 58108-6050

Please do not forget to include your name, postal address, school, and e-mail address.

Here is the first set of problems:

1. Consider the following system of three equations with three unknowns

$$\begin{aligned} *x + *y + *z &= 0, \\ *x + *y + *z &= 0, \\ *x + *y + *z &= 0, \end{aligned}$$

   where $*$ denote some numbers. Two people take turns in replacing one of the "$*$" by a number. Show that the person that starts always can choose the numbers such that the resulting final system (with numbers instead of "$*$") has a nonzero solution.

2. A piece of squared graph paper of the size 20 by 30 is given. Is it possible to draw a straight line on this paper such that it would cross 50 squares?

3. A precise square is a number a such that $a = b^2$ for some integer b. Show that the sum of the digits of a precise square cannot be equal to 5. *Hint:* What is so special about dividing a given number by 3?

4. Three circles with the centers at O_1, O_2, O_3 of the same radius intersect at the same point. Let A_1, A_2, A_3 be the other points of their intersections. Show that the triangles $O_1O_2O_3$ and $A_1A_2A_3$ are equal.

5. Suppose that 15 flags of different colors are to be displayed on 5 poles. In how many ways this can be done? (We disregard, of course, the absolute position of the flags on the poles and the practical limitations on the number of flags on a pole. We assume only that the flags on each pole are in definite order from top to bottom.)